



50X1-HUM

CONFIDENTIAL

During these movements, moreover, gyroscopic moments arise which react against the disturbing force and govern the stabilization at the first moment, so long as the load-relieving apparatus is not operating. In this case there is no need, either for the tracking system or for a super delicate designing of the external supports of the sensitive element, and the readings can be taken immediately with it. Here the frame of the gyroscopic sensitive element assumes, at the same time, the role of the tracking system.

As example of a forced stabilizer is the well-known gyroscopic hodometer the description of which is in E. Sperry's book [1]. The book also shows that, with the aid of two such Sperry stabilizers, it is possible to obtain an artificial horizon; that is, a platform able to be maintained in a position close to the horizontal.

#### Equations of Motion of a Forced Gyroscopic Horizon

One of the possible schemes is presented in the appended figure. The  $y$  axis of the Cardan wheel is directed along the longitudinal axis of the ship. The stabilizable platform can be rotated relative to the wheel around the second Cardan  $x$  axis. The angles of rotation around these axes are denoted by  $\alpha$  and  $\beta$  and are assumed to be small.  $S_x$  and  $S_y$  are the load-relieving motors. The  $\xi\eta$  plane of the true coordinate system  $\xi\eta\zeta$ , which is connected with the ship, is assumed to be horizontal.

The  $xy$  plane of the second coordinate system  $xyz$  is the plane of the platform. The housings of the four gyroscopes of natural moments  $H_x, H'_x, H_y, H'_y$  can rotate relative to the platform around the perpendiculars to their axes. So that the axes of rotation of the gyroscope remain in the  $xy$  plane. Since the housings are joined in pairs as antiparallelograms, then for small angles of precession  $\sigma, \sigma'$  of the first two gyroscopes we have approximately the relation  $\sigma' = -\sigma$ . Analogously, for the other two gyroscopes we have  $\tau = -\tau'$ . The arrows  $T_x$  and  $T_y$  in the diagram represent also the slide runners of the potentiometers, which control the load-relieving motors. The moments, transferable by the motors to the Cardan axes, are  $S_x \tau - S_y \sigma$ , where  $S_x$  and  $S_y$  are considered constant.

In order to create the regulating forces, the horizontal pendulums  $K_x$  and  $K_y$  are employed, which regulate the motors  $E_x$  and  $E_y$  placed on the precessional axes of the gyroscopes. During any movement of the ship of constant velocity  $v$  with respect to the ship course, making a constant angle  $\psi$  with the direction of the north, the inclinations of the pendulums are proportional to the projections  $p\alpha$  and  $p\beta$  of the weight  $p$  of the pendulum bob upon the axes  $x$  and  $y$ . If the speed is changed and the ship's trajectory has a radius of curvature  $R$ , then the pendulums are acted upon by other forces; namely, the forces of inertia  $pv^2/gR$ ,  $-pv/g$ . Therefore the moments generated by the regulating motors are  $K_x(\alpha + \frac{\omega v}{g})$  and  $K_y(\beta - \frac{v}{g})$ , where  $K_x$  and  $K_y$  are assumed to be constant and  $\omega = \frac{v}{R}$  is the angular velocity of circulation (ship's turning on the arc of radius  $R$ ).

As for the equations of motion, it is possible to connect the equations of moments relative to the  $x, y$  axes for the whole system and the equations of moments relative to the whole precession for the gyroscopic pairs  $H_x, H'_x$  and  $H_y, H'_y$ :

$$\begin{aligned} -B\ddot{\beta} &= T_x + S_x \tau, & A\ddot{\alpha} &= T_y - S_y \sigma, \\ T_1 + K_x(\alpha + \frac{\omega v}{g}) &= 0, & T_2 + K_y(\beta - \frac{v}{g}) &= 0. \end{aligned}$$

- 2 -

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

CONFIDENTIAL

CONFIDENTIAL

Here  $A, B$  are the moments of inertia of the whole system relative to the  $x, y$  axes and the inertial terms of form  $C_1 \ddot{\varphi}, C_2 \ddot{\tau}$  in the two latter equations are disregarded, in as much as the corresponding masses are not large;  $\Gamma_x, \Gamma_y, \Gamma_1, \Gamma_2$  are the gyroscopic moments, whose expressions may be obtained from the formulas (2.10) in the article by Ya. N. Roytenberg [2]. Designating the angular velocity of the earth's rotation by  $U$ , the earth's radius by  $R$ , the latitude of the place by  $\varphi$ , taking into consideration the various distinctions in the remaining designations, and assuming that  $H_x = H'_x, H_y = H'_y$ , then we shall find that:

$$\Gamma_x = 2H_x \sigma (U \sin \varphi + \omega) + 2H_y \dot{\tau}, \quad \Gamma_y = 2H_y \tau (U \sin \varphi + \omega) - 2H_x \dot{\sigma};$$

$$\Gamma_1 = 2H_x [U \cos \varphi \cos \psi + \dot{\alpha} - (U \sin \varphi + \omega) \beta],$$

$$\Gamma_2 = 2H_y [U \cos \varphi \sin \psi + \frac{v}{R} + \dot{\beta} + (U \sin \varphi + \omega) \alpha].$$

Substituting in the equations of motion and disregarding the products of  $U \sin \varphi$  by  $\alpha, \beta, \sigma, \tau$  we then obtain:

$$-B \ddot{\beta} = 2H_x \omega \sigma + 2H_y \dot{\tau} + S_x \dot{\tau}, \quad A \ddot{\alpha} = 2H_y \omega \tau - 2H_x \dot{\sigma} - S_y \sigma, \quad (1)$$

$$2H_x (U \cos \varphi \cos \psi + \dot{\alpha} - \omega \beta) + K_x (\alpha + \frac{\omega v}{g}) = 0, \quad (2)$$

$$2H_y (U \cos \varphi \sin \psi + \frac{v}{R} + \dot{\beta} + \omega \alpha) + K_y (\beta - \frac{v}{g}) = 0.$$

Hence it is seen that, if a ship moves with constant velocity with respect to a constant course ( $v = \text{const}, \varphi = \text{const}, \dot{v} = \omega = 0$ ), then the coordinates  $\alpha, \beta, \sigma, \tau$  vary aperiodically and tend to a constant limiting value  $\alpha^*, \beta^*, \sigma^*, \tau^*$ , where:

$$\alpha^* = -\frac{2H_x U \cos \varphi \cos \psi}{K_x}, \quad \beta^* = -\frac{2H_y (U \cos \varphi \sin \psi + \frac{v}{R})}{K_y} \quad (3)$$

quantities are velocity deviations; they are small if  $K_x, K_y$  are sufficiently large. For  $\dot{v} \neq 0$  and  $\omega \neq 0$  the apparatus receives several ballistic deviations which can be studied by the usual methods [3].

#### Possibility of Obtaining an 84-Minute Period and of Compensating for Ballistic Deviations

We shall modify the preceding scheme by uniting the pendulum  $K_x$  with the gyroscopic  $H_y, H'_y$ , and the pendulum  $K_y$  with the gyroscopic  $H_x, H'_x$ . Equations (1) are not modified, but in equations (2) the expressions  $K_x (\alpha + \frac{\omega v}{g})$  and  $K_y (\beta - \frac{v}{g})$  will vary slightly in places where we effect connections in such a manner that the second expression has the minus sign. Assuming that:

$$\frac{K_y}{2H_x} = \frac{K_x}{2H_y} = k, \quad (4)$$

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

CONFIDENTIAL

CONFIDENTIAL

we obtain instead of (2)

$$\begin{aligned} \dot{\alpha} - (k + \omega)\beta &= -U \cos \varphi \cos \psi - \frac{kv}{g}, \\ \dot{\beta} + (k + \omega)\alpha &= -U \cos \varphi \sin \psi - \frac{v}{R} - \frac{k\omega v}{g}; \end{aligned} \quad (5)$$

or by setting

$$z = \alpha + i\beta, \quad (6)$$

and multiplying by 1 and 1 and then adding, we obtain:

$$\dot{z} + i(k + \omega)z = -U \cos \varphi e^{i\psi} - \frac{iv}{R} - \frac{k(v + i\omega v)}{g} \quad (7)$$

We are then led to a new unknown:

$$\zeta = z - z^*, \quad (8)$$

where the quantity

$$z^* = \frac{(iU \cos \varphi e^{i\psi} - \frac{v}{R})}{k} \quad (9)$$

is the velocity deviation; we note that  $\dot{\psi} = -\omega$  and we write

$$v = \sqrt{\frac{g}{R}} = 1,241 \cdot 10^{-3} \text{ sec}^{-1}; \quad (10)$$

hence, we obtain the following transformation equation

$$\dot{\zeta} + i(k + \omega)\zeta = -\frac{g}{g} \left(1 - \frac{v^2}{k^2}\right) (\dot{\psi} + i\omega v). \quad (11)$$

If we select the parameters such that  $k = \pm v$ , then the equation becomes homogeneous. This indicates that the apparatus in this case does not possess ballistic deviations. Thus for the given system compensation is possible according to M. Shuler, since the system acts during compensation as a gyroscopic pendulum with a period of precession:

$$T = \frac{2\pi}{v} = 84,4 \text{ minutes}. \quad (12)$$

It does not present any difficulties to provide for dampening of the free precession. In order to do this, it is necessary to combine both of the discussed variants of the system by joining each of the pendulums with both gyroscopic pairs and by selecting in a suitable manner the transmission ratios of signals.

#### BIBLIOGRAPHY

1. E. Sperry. Applied Gyrodynamics, New York, pp 125-128, 1933
2. Ya. N. Roytenburg, "Poly-gyroscopic Vertical" Applied Mathematics and Mechanics, X, No 1, 1946
3. B. V., Bulgakov, Applied Theory of Gyroscopes, M-L, 1939

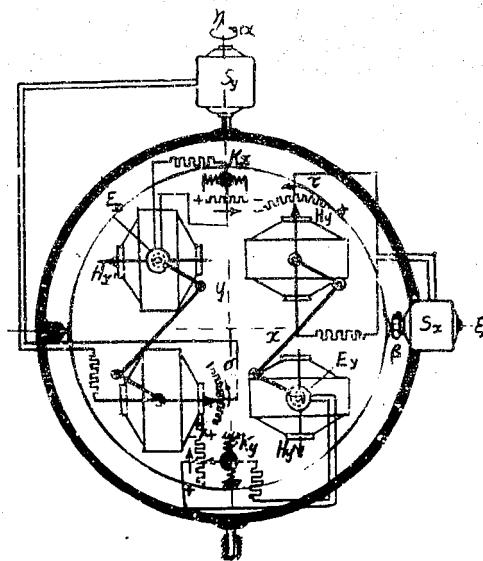
[Appended figure follows]

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

CONFIDENTIAL



- E N D -

- 5 -

CONFIDENTIAL